MCS287 Exam 1

Serial #:

This exam is a closed-book, individual-work exam. It is also closed notes, with the exception of one 8.5 × 11 inch handwritten sheet of notes (both sides OK). Please write your name only on this page. Do not turn the page until instructed, in order that everyone may have the same time. Then, be sure to look at all problems before deciding which one to do first. Some problems are easier than others, so plan your time accordingly. You have 50 minutes to work.

Write the answer to each problem on the page on which that problem appears. You may use the back of that same sheet if you need additional space. You may also request additional paper, which should be labeled with your test number and the problem number.

Name: ________________________________

Problem	Page	Possible	Score
1	2	15	
2	3	12	
3	4	12	
4	5	12	
5	6	12	
Total		63	

1. [**15 Points**] Circle all free occurrences of variables and put square boxes around all bound occurrences of variables in the following Scheme expressions:

```
((lambda (x)
   (x y))
 x)

(lambda (x)
  (lambda (y)
    x))

(lambda (x)
  ((lambda (y)
     z)
   (x y)))

(let ((x y)
      (y x))
  (x y))

(letrec ((x (lambda (z)
              y))
         (y (lambda (u)
              x)))
  (x y))
```

2. [**12 Points**] Write a single expression of the pure lambda calculus in which

- `x` occurs free and also occurs bound
- `y` occurs free but not bound
- `z` occurs bound but not free

3. [**12 Points**] Underline the left parentheses in the following expressions that start β-redexes. You should consider both the top-level expressions and their subexpressions when looking for β-redexes. There may be more than one β-redex in the same top-level expression. Don't count β-redexes that only appear after another β-reduction has been done, just those that are already here:

```
((lambda (x)
   (x (lambda (y)
        y)))
 (lambda (z)
   z))

(lambda (x)
  (x ((lambda (y)
        y)
      (lambda (z)
        z))))

((lambda (x)
   ((lambda (y)
      y)
    x))
 (lambda (z)
   z))

(lambda (x)
  ((lambda (y)
     (y ((lambda (z)
           y)
          x)))
   x))
```

4. [**12 Points**] Perform one β-reduction on each of the following β-redexes:

```
((lambda (x)
   (lambda (y)
     (y x)))
 (lambda (z)
   z))
```

```
((lambda (x)
   (lambda (x)
     (x x)))
 (lambda (z)
   z))
```

```
((lambda (x)
   (lambda (y)
     (y y)))
 (lambda (z)
   z))
```

```
((lambda (x)
   (lambda (y)
     (y x)))
 (lambda (z)
   y))
```

5. [**12 Points**] For each of the following expressions, mark the first β-redex that would be reduced by the leftmost reduction strategy, by underlining its opening left parenthesis. Also mark the first β-redex that would be reduced by the applicative-order strategy, by underlining its closing right parenthesis.

```
((lambda (x)
   x)
 ((lambda (y)
    y)
  z))
```

```
(((lambda (x)
    x)
  z)
 ((lambda (y)
    y)
  z))
```

```
((x
  (lambda (y)
    ((lambda (z)
       z)
     w)))
 ((lambda (u)
    u)
  v))
```